

1. Sample spaces.

- a) $S = \{HH, HT, TH, TT\}$ All of the outcomes are equally likely to occur.
- b) $S = \{0, 1, 2, 3\}$ All outcomes are not equally likely. A family of 3 is more likely to have, for example, 2 boys than 3 boys. There are three equally likely outcomes that result in 2 boys (BBG, BGB, and GBB), and only one that results in 3 boys (BBB).
- c) $S = \{H, TH, TTH, TTT\}$ All outcomes are not equally likely. For example the probability of getting heads on the first try is $\frac{1}{2}$. The probability of getting three tails is $\left(\frac{1}{2}\right)^3 = \frac{1}{8}$.
- d) $S = \{1, 2, 3, 4, 5, 6\}$ All outcomes are not equally likely. Since you are recording only the larger number of two dice, 6 will be the larger when the other die reads 1, 2, 3, 4, or 5. The outcome 2 will only occur when the other die shows 1 or 2.

3. Homes.

Construct a Venn diagram of the disjoint outcomes.

$$\begin{aligned} \text{a) } P(\text{pool} \cup \text{garage}) &= P(\text{pool}) + P(\text{garage}) - P(\text{pool} \cap \text{garage}) \\ &= 0.64 + 0.21 - 0.17 = 0.68 \end{aligned}$$

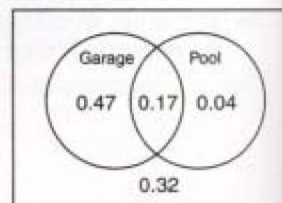
Or, from the Venn: $0.47 + 0.17 + 0.04 = 0.68$

$$\text{b) } P(\text{neither}) = 1 - P(\text{pool} \cup \text{garage}) = 1 - 0.68 = 0.32$$

Or, from the Venn: 0.32 (the region outside the circles)

$$\text{c) } P(\text{pool} \cap \text{no garage}) = P(\text{pool}) - P(\text{pool} \cap \text{garage}) = 0.21 - 0.17 = 0.04$$

Or, from the Venn: 0.04 (the region inside pool circle, yet outside garage circle)

**5. Amenities.**

Construct a Venn diagram of the disjoint outcomes.

$$\begin{aligned} \text{a) } P(\text{TV} \cap \text{no refrigerator}) &= P(\text{TV}) - P(\text{TV} \cap \text{refrigerator}) \\ &= 0.52 - 0.21 = 0.31 \end{aligned}$$

Or, from the Venn: 0.31

(the region inside the TV circle, yet outside the Fridge circle)

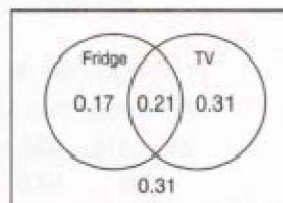
$$\begin{aligned} \text{b) } P(\text{refrigerator} \cup \text{TV, but not both}) &= \\ &= [P(\text{refrigerator}) - P(\text{refrigerator} \cap \text{TV})] + [P(\text{TV}) - P(\text{refrigerator} \cap \text{TV})] \\ &= [0.38 - 0.21] + [0.52 - 0.21] = 0.48 \end{aligned}$$

This problem is much easier to visualize using the Venn diagram. Simply add the probabilities in the two regions for Fridge only and TV only.

$$P(\text{refrigerator} \cup \text{TV, but not both}) = 0.17 + 0.31 = 0.48$$

$$\begin{aligned} \text{c) } P(\text{neither TV nor refrigerator}) &= 1 - P(\text{either TV} \cup \text{refrigerator}) \\ &= 1 - [P(\text{TV}) + P(\text{refrigerator}) - P(\text{TV} \cap \text{refrigerator})] \\ &= 1 - [0.52 + 0.38 - 0.21] \\ &= 0.31 \end{aligned}$$

Or, from the Venn: 0.31 (the region outside the circles)



7. First lady.

- a) $P(\text{Laura Bush}) = \frac{518}{1005} \approx 0.515$ b) $P(\text{younger than 50 years}) = \frac{217 + 416}{1005} \approx 0.630$
- c) $P(\text{younger than 50} \cap \text{Hillary Clinton}) = \frac{135 + 158}{1005} \approx 0.292$
- d) $P(\text{younger than 50} \cup \text{Hillary Clinton})$
 $= P(\text{younger than 50}) + P(\text{Clinton}) - P(\text{younger than 50} \cap \text{Clinton})$
 $= \frac{217 + 416}{1005} + \frac{437}{1005} - \frac{135 + 158}{1005} \approx 0.773$

13. First lady, take 2.

- a) $P(\text{between 18 and 29} \cap \text{Clinton}) = \frac{135}{1005} \approx 0.134$
- b) $P(\text{Clinton} \mid \text{between 18 and 29}) = \frac{135}{217} \approx 0.622$
- c) $P(\text{between 18 and 29} \mid \text{Clinton}) = \frac{135}{437} \approx 0.309$
- d) $P(\text{over 65} \mid \text{Bush}) = \frac{92}{518} \approx 0.178$ e) $P(\text{Bush} \mid \text{over 65}) = \frac{92}{167} \approx 0.551$

15. Sick kids.

Having a fever and having a sore throat are not independent events, so:

$$P(\text{fever and sore throat}) = P(\text{Fever}) P(\text{Sore Throat} \mid \text{Fever}) = (0.70)(0.30) = 0.21$$

The probability that a kid with a fever has a sore throat is 0.21.

21. Eligibility.

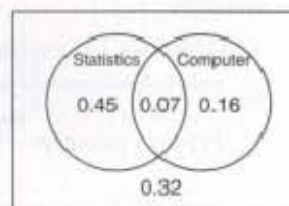
Construct a Venn diagram of the disjoint outcomes.

a)

$$\begin{aligned} P(\text{eligibility}) &= P(\text{statistics}) + P(\text{computer science}) - P(\text{both}) \\ &= 0.52 + 0.23 - 0.07 \\ &= 0.68 \end{aligned}$$

68% of students are eligible for BioResearch, so $100 - 68 = 32\%$ are ineligible.

From the Venn, the region outside the circles represents those students who have taken neither course, and are therefore ineligible for BioResearch.



b)

$$P(\text{computer science} \mid \text{statistics}) = \frac{P(\text{computer science} \cap \text{statistics})}{P(\text{statistics})} = \frac{0.07}{0.52} = 0.135$$

From the Venn, consider only the region inside the Statistics circle. The probability of having taken computer science is 0.07 out of a total of 0.52 (the entire Statistics circle).

c) Taking the two courses are not disjoint events, since they have outcomes in common. In fact, 7% of juniors have taken both courses.

d) Taking the two courses are not independent events. The overall probability that a junior has taken a computer science is 0.23. The probability that a junior has taken a computer course given that he or she has taken a statistics course is 0.135. If taking the two courses were independent events, these probabilities would be the same.

25. Cards.

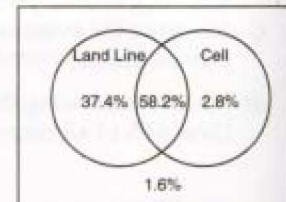
Yes, getting an ace is independent of the suit when drawing one card from a well shuffled deck. The overall probability of getting an ace is $4/52$, or $1/13$, since there are 4 aces in the deck. If you consider just one suit, there is only 1 ace out of 13 cards, so the probability of getting an ace given that the card is a diamond, for instance, is $1/13$. Since the probabilities are the same, getting an ace is independent of the suit.

27. First lady, final visit.

- a) Yes, since they share no outcomes. No one is both under 30 and over 65.
- b) No, since knowing that one event is true drastically changes the probability of the other. The probability of a respondent chosen at random being under 30 is almost 22%. The probability of being under 30, given that the respondent is over 65 is 0.
- c) No, since the events share outcomes. There were 65 respondents who were over 65 and chose Clinton.
- d) No, since knowing that one event is true drastically changes the probability of the other. Over 43% of all respondents chose Clinton, but only 39% of those over 65 did.

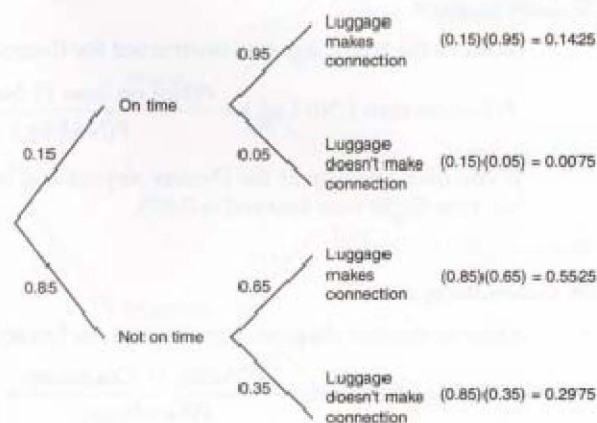
31. Phone service.

- a) Since 2.8% of U.S. adults have only a cell phone, and 1.6% have no phone at all, polling organizations can reach $100 - 2.8 - 1.6 = 96.5\%$ of U.S. adults.
- b) Using the Venn diagram, about 96.5% of U.S. adults have a land line. The probability of a U.S. adults having a land line given that they have a cell phone is $58.2/(58.2+2.8)$ or about 95.4%. It appears that having a cell phone and having a land line are independent, since the probabilities are roughly the same.

**35. Luggage.**

Organize using a tree diagram.

- a) No, the flight leaving on time and the luggage making the connection are not independent events. The probability that the luggage makes the connection is dependent on whether or not the flight is on time. The probability is 0.95 if the flight is on time, and only 0.65 if it is not on time.



b)

$$\begin{aligned}
 P(\text{Luggage}) &= P(\text{On time} \cap \text{Luggage}) + P(\text{Not on time} \cap \text{Luggage}) \\
 &= (0.15)(0.95) + (0.85)(0.65) \\
 &= 0.695
 \end{aligned}$$

43. Drunks.

Organize the information into a tree diagram.

a) $P(\text{Detain} \mid \text{Not Drinking}) = 0.2$

b)

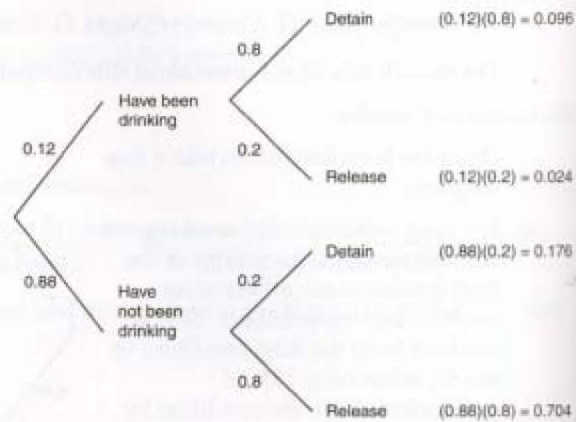
$$\begin{aligned}
 &P(\text{Detain}) \\
 &= P(\text{Drinking} \cap \text{Det.}) \\
 &\quad + P(\text{Not Drinking} \cap \text{Det.}) \\
 &= (0.12)(0.8) + (0.88)(0.2) \\
 &= 0.272
 \end{aligned}$$

c)

$$\begin{aligned}
 P(\text{Drunk} \mid \text{Det.}) &= \frac{P(\text{Drunk} \cap \text{Det.})}{P(\text{Detain})} \\
 &= \frac{(0.12)(0.8)}{(0.12)(0.8) + (0.88)(0.2)} \\
 &\approx 0.353
 \end{aligned}$$

d)

$$\begin{aligned}
 P(\text{Drunk} \mid \text{Release}) &= \frac{P(\text{Drunk} \cap \text{Release})}{P(\text{Release})} \\
 &= \frac{(0.12)(0.2)}{(0.12)(0.2) + (0.88)(0.8)} \\
 &\approx 0.033
 \end{aligned}$$



45. Dishwashers.

Organize the information in a tree diagram.

$$\begin{aligned}
 &P(\text{Chuck} \mid \text{Break}) \\
 &= \frac{P(\text{Chuck} \cap \text{Break})}{P(\text{Break})} \\
 &= \frac{(0.3)(0.03)}{(0.4)(0.01) + (0.3)(0.01) + (0.3)(0.03)} \\
 &= 0.563
 \end{aligned}$$

If you hear a dish break, the probability that Chuck is on the job is approximately 0.563.

